Voltage Stability

P. Kundur, Power System Stability and Control 1st Edition

Graduate School of Energy Convergence

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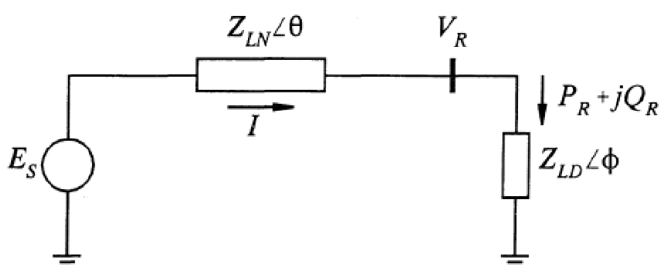
Basic Concepts Related to Voltage Stability

- Principal factors of voltage collapse
  - Generator reactive power/voltage control limits
  - Load and reactive compensation devices' characteristics
  - Action of voltage control devices (such as ULTCs)
- Transmission system characteristics
  - Review of the simple system considered in CH 2
  - Current, voltage and power are:

$$I = \frac{1}{\sqrt{F}} \frac{E_S}{Z_{LN}} \qquad (14.1)$$

$$V_R = \frac{1}{\sqrt{F}} \frac{Z_{LD}}{Z_{LN}} E_S \qquad (14.2)$$

$$P_R = \frac{Z_{LD}}{F} \left(\frac{E_S}{Z_{LN}} \right)^2 \cos\phi \qquad (14.3)$$



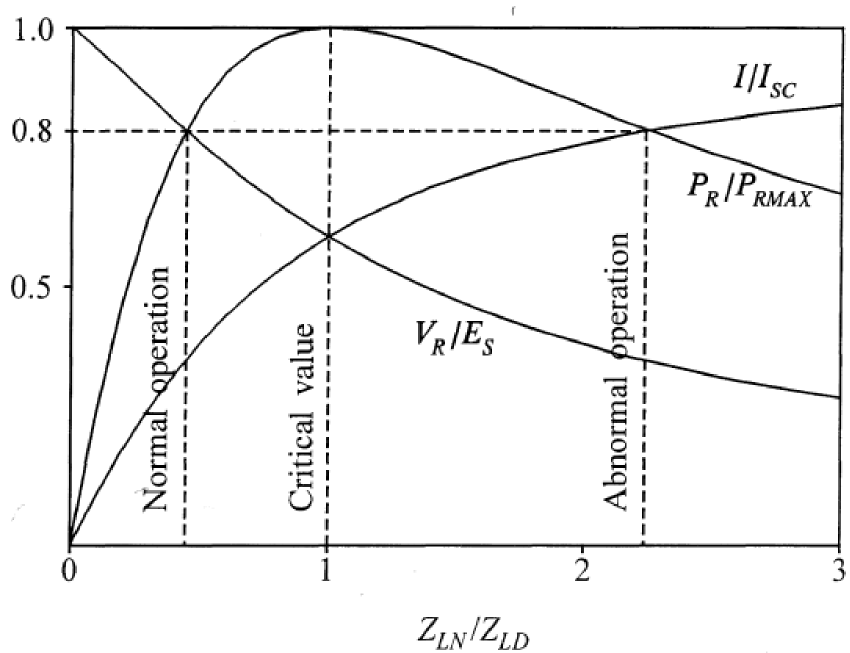
(a) Schematic diagram

Figure 14.1 Characteristics of a simple radial system

$$\boxed{F = 1 + \left(\frac{Z_{LD}}{Z_{LN}} \right)^2 + 2 \left(\frac{Z_{LD}}{Z_{LN}} \right) \cos(\theta - \phi)}$$

Basic Concepts Related to Voltage Stability

- Transmission system characteristics (Cont'd)
  - Plots of I, V_R, and P_R as a function of load demand



$\tan(\theta) = 10, \cos(\phi) = 0.95$

$\phi = 18.2°$
$\theta = 84.3°$
$I_{SC} = E_S/Z_{LN}$

(b) Receiving end voltage, current and power as a function of load demand

Figure 14.1 Characteristics of a simple radial system

Basic Concepts Related to Voltage Stability

- Transmission system characteristics (Cont'd)
  - Relationship between V_R and P_R (More traditional method)

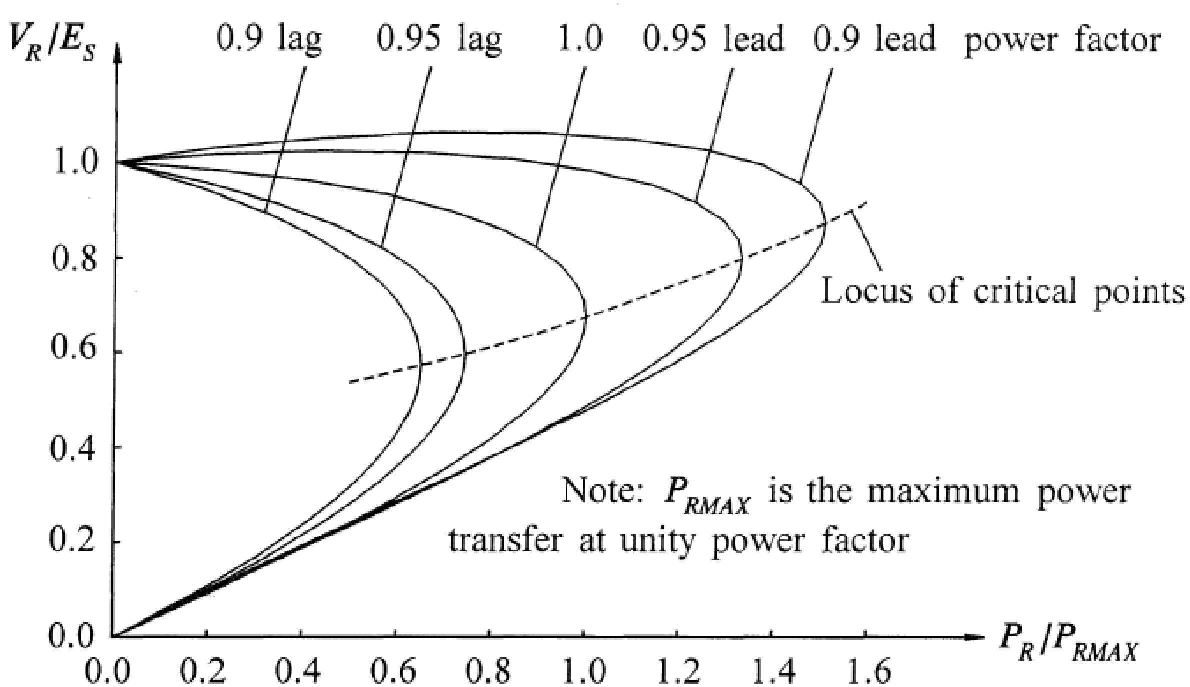


Figure 14.2 The V_R-P_R characteristics of the system of Figure 14.1

Basic Concepts Related to Voltage Stability

- Transmission system characteristics (Cont'd)
  - Test system

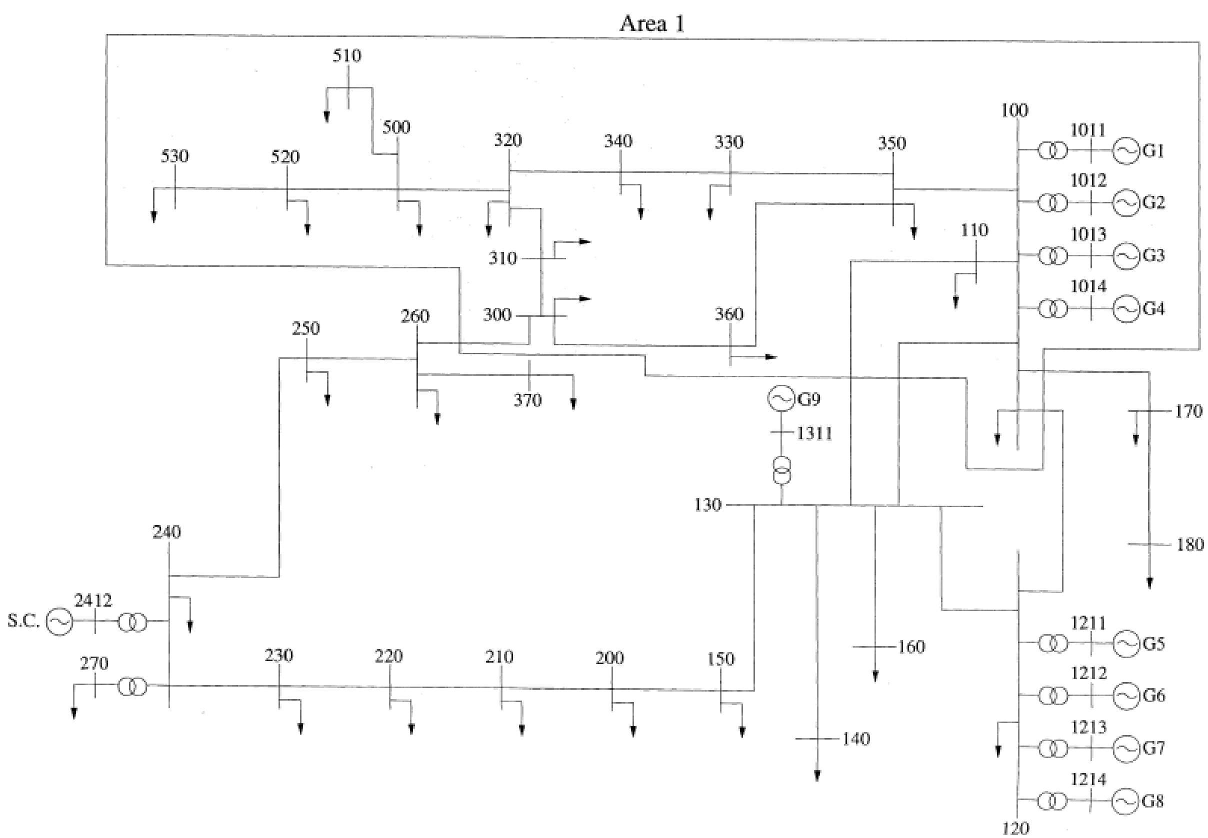


Figure 14.3 A 39-bus, 10-machine test system

- Transmission system characteristics (Cont'd)

  V variation at bus 530 as a function of total P load in area 1

  - Curve is produced by using a series of power flow solutions
  - Assumption
    - Power factor is kept constant
    - P and Q load are independent of V

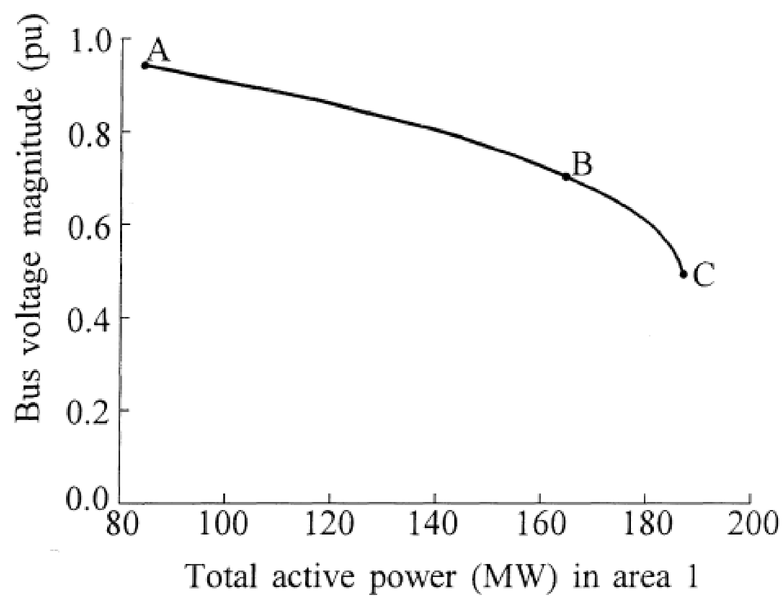


Figure 14.4 The *V*-*P* curve at bus 530 of the system shown in Figure 14.3

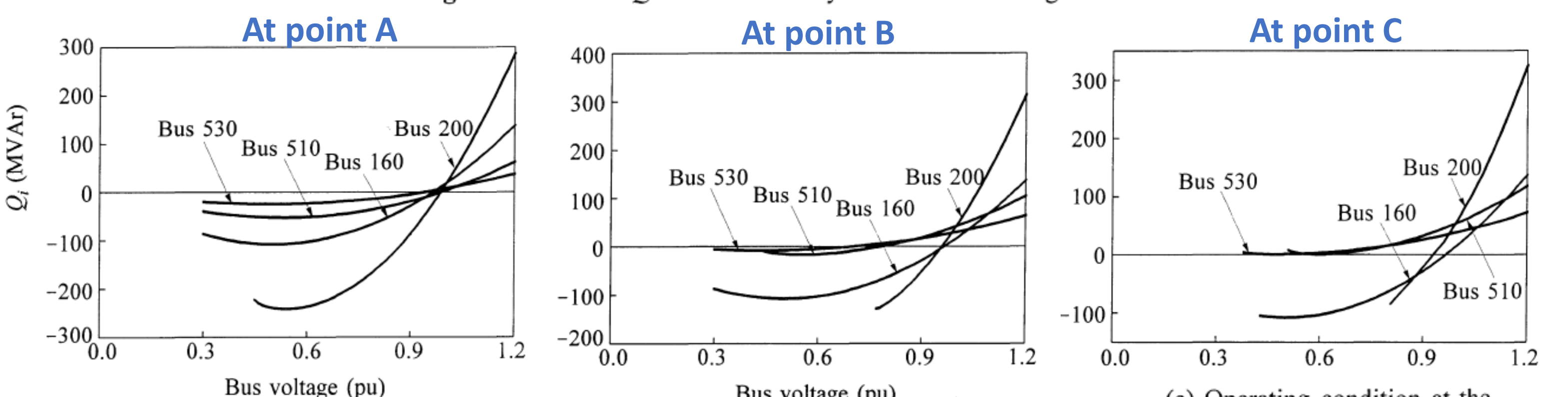
Basic Concepts Related to Voltage Stability

- Transmission system characteristics (Cont'd)

Q-V curves at buses 160, 200, 510, and 530

- Calculated as PV buses
- A point where $dQ/dV = 0$ represents the voltage stability limit
- Principal causes of voltage instability
  - Too high load, Too far between voltage sources and the load centers, Too low voltage source, Insufficient load reactive compensation

Figure 14.5 The Q-V curves for system shown in Figure 14.3



(a) Base case, with a load of 84.5 MW, 53.3 MVAr in area

(b) Operating condition near the critical operating point, with a load of 164.4 MW, 103.7 MVAr in area 1

(c) Operating condition at the critical operating point, with a load of 186.9 MW, 117.9 MVAr in area 1

Basic Concepts Related to Voltage Stability

- Generator characteristics
  - Field current is automatically limited by an overexcitation limiter
  - Armature current limit is realized manually by operators responding to alarms
  - Curve 2: When excitation is limited
  - Curve 1: Voltage regulated

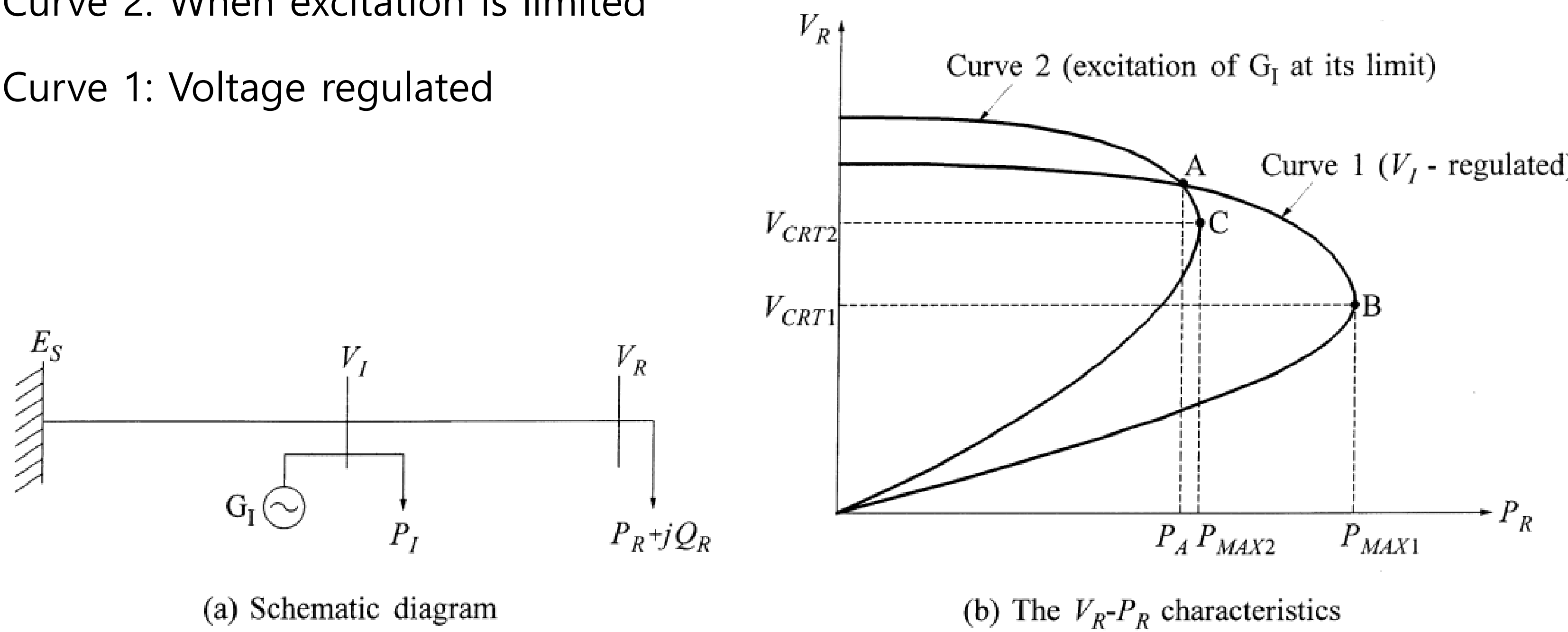


Figure 14.6 Impact of loss of regulation of intermediate bus voltage

Basic Concepts Related to Voltage Stability

- Load characteristics
  - Voltages are determined by composite characteristics of transmission system and loads
  - When ULTCs reach the end of their tap change, voltages begin to drop
    - Residential P and Q loads will drop with voltage
    - Industrial loads (induction motors) will change little
    - However, the capacitor will supply less Q, thereby causing Q load increase
  - Voltages below 80%~90%, some induction motors may draw high Q

Basic Concepts Related to Voltage Stability

- Characteristics of reactive compensating devices
  - Shunt capacitors
    - $Q \propto V^2$
    - Beyond certain level of compensation, stable operation is unattainable
  - Regulated shunt compensation
    - SVS: NO voltage control or instability problems within the regulating range
    - Synchronous condenser: Continues to supply Q, down to relatively low V
  - Series capacitors
    - $Q \propto I^2$
    - Reduce both the characteristic impedance and the electrical length of the line

Voltage Collapse

- Typical scenario of voltage collapse

Large generating units being out of service

↓

Some EHV lines are heavily loaded
(Q resources become at a minimum)

↓

Loss of heavily loaded line

↓

Considerable reduction of voltage

↓

Load reduction and stabilizing (i.e., AVR)

↓

Resulting additional Q cause increased V drop across each inductive element

↓

ULTC would restore distribution voltages to prefault levels in about 2~4 minutes

↓

Increment in load on EHV lines would increase XI^2 and RI^2 losses

↓

Q of generators would increase and eventually hit their limit

↓

Field current limited and the terminal voltage would drop

↓

Armature current increase due to fixed P and further limit Q

↓

Overload more generators

↓

Reduced effectiveness of shunt compensators at low voltages

Voltage Collapse

- Classification of voltage stability
  - Large disturbance voltage stability
    - Concerned with a systems ability to control voltages following large disturbances
    - Must be examined by using nonlinear dynamic analysis
  - Small disturbance voltage stability
    - Concerned with a system's ability to control voltages following small perturbations
    - Can be examined by using steady-state analysis
    - Necessary to define the region of voltage level considered acceptable

Voltage Stability Analysis

- Examination of two aspects
  - Proximity to voltage instability: How close is the system to voltage instability?
  - Mechanism of voltage instability: How and why does instability occur?

- Modelling requirements
  - Loads
  - Generators and their excitation controls
  - Static var systems
  - Automatic generation control
  - Protection and controls

Voltage Stability Analysis

- Dynamic analysis
  - Similar to transient stability analysis in CH 13
  - Overall system equations may be expressed as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{V}) \tag{14.4}$$

  - And a set of algebraic equations:

$$\mathbf{I}(\mathbf{x}, \mathbf{V}) = \mathbf{Y}_N \mathbf{V} \tag{14.5}$$

  - With a set of known initial conditions ($\mathbf{x_0}$, $\mathbf{V_0}$), where

$\mathbf{x}$ = state vector of the system
$\mathbf{V}$ = bus voltage vector
$\mathbf{I}$ = current injection vector
$\mathbf{Y}_N$ = network node admittance matrix

Voltage Stability Analysis

- Dynamic analysis (Cont'd)
  - $\mathbf{Y_N}$ change as a function of bus voltage and time (ULTC, phase shift control)
  - $\mathbf{I}$ is a function of $\mathbf{x}$ and $\mathbf{V}$ (generating units, nonlinear static loads, motors, etc.)
  - (14.4) and (14.5) can be solved in time-domain by using
    - Any of numerical integration methods in CH 13
    - Network power flow analysis methods in CH 6

Voltage Stability Analysis

- Static analysis
  - Captures *snapshots* of system conditions at various time frames
  - Time derivatives of the state variables ($\dot{\mathbf{x}}$) are assumed to be zero

V-Q sensitivity analysis

- Network may be expressed in the following linearized form:

$$\begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{P\theta} & \mathbf{J}_{PV} \\ \mathbf{J}_{Q\theta} & \mathbf{J}_{QV} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta \mathbf{V} \end{bmatrix} \qquad (14.6)$$

where

$\Delta\mathbf{P}$ = incremental change in bus real power
$\Delta\mathbf{Q}$ = incremental change in bus reactive power injection
$\Delta\boldsymbol{\theta}$ = incremental change in bus voltage angle
$\Delta\mathbf{V}$ = incremental change in bus voltage magnitude

Voltage Stability Analysis

- Static analysis (Cont'd)

V-Q sensitivity analysis (Cont'd)

- For each device when $\dot{\mathbf{x}} = \mathbf{0}$ may be expressed as follows:

$$\begin{bmatrix} \Delta\mathbf{P}_d \\ \Delta\mathbf{Q}_d \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{V}_d \\ \Delta\boldsymbol{\theta}_d \end{bmatrix} \qquad (14.7)$$

where

$\Delta\mathbf{P}_d$ = incremental change in device real power output
$\Delta\mathbf{Q}_d$ = incremental change in device reactive power output
$\Delta\mathbf{V}_d$ = incremental change in device voltage magnitude
$\Delta\boldsymbol{\theta}_d$ = incremental change in device voltage angle

- Let $\Delta\mathbf{P} = 0$, then

$$\Delta\mathbf{Q} = \mathbf{J}_R \Delta\mathbf{V} \qquad (14.8)$$

where

$$\mathbf{J}_R = [\mathbf{J}_{QV} - \mathbf{J}_{Q\theta}\mathbf{J}_{P\theta}^{-1}\mathbf{J}_{PV}] \qquad (14.9)$$

Voltage Stability Analysis

- Static analysis (Cont'd)

V-Q sensitivity analysis (Cont'd)

- We may write

$$\Delta \mathbf{V} = \mathbf{J}_R^{-1} \Delta \mathbf{Q} \quad (14.10)$$

- Matrix $\mathbf{J}_R^{-1}$ is the reduced V-Q Jacobian
- ith diagonal element is the V-Q sensitivity at bus i
- Magnitude of V-Q sensitivity:
  - Positive: Stable operation
  - Smaller value: More stable
- V-Q relationship is nonlinear
  - Magnitudes of the sensitivities for different system conditions do not provide a direct measure of the relative degree of stability

Voltage Stability Analysis

- Static analysis (Cont'd)

Q-V modal analysis

- Voltage stability characteristics of the system can be identified by computing the eigenvalues and eigenvectors of the reduced Jacobian matrix $\mathbf{J_R}$

$$\mathbf{J}_R = \boldsymbol{\xi}\boldsymbol{\Lambda}\boldsymbol{\eta} \quad (14.11)$$

where $\boldsymbol{\xi}$ = right eigenvector matrix of $\mathbf{J}_R$
$\boldsymbol{\eta}$ = left eigenvector matrix of $\mathbf{J}_R$
$\boldsymbol{\Lambda}$ = diagonal eigenvalue matrix of $\mathbf{J}_R$

- From (14.11), $\mathbf{J}_R^{-1} = \boldsymbol{\xi}\boldsymbol{\Lambda}^{-1}\boldsymbol{\eta}$ (14.12)

Each λ_i and corresponding ξ_i and η_i define ith mode of Q- V response

- Substituting (14.10) gives

$$\Delta\mathbf{V} = \boldsymbol{\xi}\boldsymbol{\Lambda}^{-1}\boldsymbol{\eta}\Delta\mathbf{Q} \quad (14.13)$$

$$\Delta\mathbf{V} = \sum_i \frac{\boldsymbol{\xi}_i\boldsymbol{\eta}_i}{\lambda_i}\Delta\mathbf{Q} \quad (14.14)$$

where

$\boldsymbol{\xi}_i$ is the i^{th} column right eigenvector and $\boldsymbol{\eta}_i$ the i^{th} row left eigenvector of $\mathbf{J}_R$.

Voltage Stability Analysis

- Static analysis (Cont'd)

Q-V modal analysis (Cont'd)

- Since $\boldsymbol{\xi}^{-1} = \boldsymbol{\eta}$, (14.13) may be written as:

$$\boldsymbol{\eta}\Delta\mathbf{V} = \boldsymbol{\Lambda}^{-1}\boldsymbol{\eta}\,\Delta\mathbf{Q}$$

$$\mathbf{v} = \boldsymbol{\Lambda}^{-1}\mathbf{q} \qquad (14.15)$$

where

$\mathbf{v} = \boldsymbol{\eta}\Delta\mathbf{V}$ is the vector of modal voltage variations

$\mathbf{q} = \boldsymbol{\eta}\Delta\mathbf{Q}$ is the vector of modal reactive power variations

- For comparison between (14.10) and (14.15)

$$\Delta\mathbf{V} = \mathbf{J}_R^{-1}\Delta\mathbf{Q} \qquad (14.10)$$

Voltage Stability Analysis

- Static analysis (Cont'd)

Q-V modal analysis (Cont'd)

- For ith mode, we have

$$\mathbf{v}_i = \frac{1}{\lambda_i}\mathbf{q}_i \qquad (14.16)$$

- If $\lambda_i > 0$, the system is voltage stable
- If $\lambda_i < 0$, the system is voltage unstable
- The smaller the magnitude of λ_i, the closer the ith modal voltage is to being unstable
- $\lambda_i = 0$ → ith modal voltage collapse

Voltage Stability Analysis

- Static analysis (Cont'd)

Q-V modal analysis (Cont'd)

- For example, for a 3-bus system,

$$\begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} 1/\lambda_1 & & \\ & 1/\lambda_1 & \\ & & 1/\lambda_1 \end{bmatrix} \begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{bmatrix} \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}$$

- For mode 1:

$$(\eta_{11}\Delta V_1 + \eta_{12}\Delta V_2 + \eta_{13}\Delta V_3) = \frac{1}{\lambda_1}(\eta_{11}\Delta Q_1 + \eta_{12}\Delta Q_2 + \eta_{13}\Delta Q_3)$$

or

$$\mathbf{q_1} = \frac{1}{\lambda_1}\mathbf{v_1}$$

Voltage Stability Analysis

- Static analysis (Cont'd)

Q-V modal analysis (Cont'd)

- Let $\Delta\mathbf{Q} = \mathbf{e_k}$, where $\mathbf{e_k}$ has all zero elements except for the kth element = 1. Then,

$$\Delta\mathbf{V} = \sum_i \frac{\eta_{ik}\xi_i}{\lambda_i}$$

where η_{ik} is the kth element of $\boldsymbol{\eta}_i$

- The V-Q sensitivity at bus k is given by

$$\frac{\partial V_k}{\partial Q_k} = \sum_i \frac{\xi_{ki}\eta_{ik}}{\lambda_i} \qquad (14.17)$$

→ Cannot identify individual voltage collapse modes

→ Instead, provide combined effect of all modes

- The magnitude of the eigenvalues can provide a relative measure of the proximity to instability
- Eigenvalues do not provide an absolute measure because of the nonlinearity of the problem

Voltage Stability Analysis

- Static analysis (Cont'd)

Bus participation factors

- Participation of bus k in mode i is given by the bus participation factor:

$$P_{ki} = \xi_{ki}\eta_{ik} \tag{14.18}$$

→ Determines that the contribution of λ_i to the V- Q sensitivity at bus k

- Sum of all the bus participations for each mode = 1
- Indicates the effectiveness of remedial actions applied at that bus in stabilizing the mode

Voltage Stability Analysis

- Static analysis (Cont'd)

Bus participation factors (Cont'd)

- Type 1: Localized mode
  - Has very few buses with large participations
  - Other buses with close to zero participations
  - Occurs if a single load bus is connected to very strong network through a long transmission line
- Type 2: Non-localized mode
  - Has many buses with small but similar degrees of participations
  - Rest of the buses with close to zero participations
  - Occurs when a region within a large system is loaded up and the main Q support is exhausted

Voltage Stability Analysis

- Static analysis (Cont'd)

Branch participation factors

- Let $\mathbf{q}$ has all elements = 0 except for the ith element = 1
- From (14.15), corresponding vector of Q variation is

$$\Delta \mathbf{Q}^{(i)} = \boldsymbol{\eta}^{-1}\mathbf{q} = \boldsymbol{\xi}\mathbf{q} = \boldsymbol{\xi}_i \qquad (14.19)$$

- Assume that all the right eigenvectors are normalized so that

$$\sum_j \xi_{ji}^2 = 1 \qquad (14.20)$$

- With

$$\Delta \mathbf{V}^{(i)} = \frac{1}{\lambda_i}\Delta \mathbf{Q}^{(i)} \qquad (14.21) \qquad\qquad \Delta \boldsymbol{\theta}^{(i)} = -\mathbf{J}_{P\theta}^{-1}\mathbf{J}_{PV}\Delta \mathbf{V}^{(i)} \qquad (14.22)$$

- With the angle and voltage variations for both ends are known, the linearized change in branch Q loss can be calculated

Voltage Stability Analysis

- Static analysis (Cont'd)

Branch participation factors

- Participation factor:

$$P_{ji} = \frac{\Delta Q_{loss} \text{ for branch } j}{\text{maximum } \Delta Q_{loss} \text{ for all branches}} \tag{14.23}$$

→ High value means either weak links or heavily loaded

Generator participation factors

- Given by:

$$P_{mi} = \frac{\Delta Q_m \text{ for machine } m}{\text{maximum } \Delta Q \text{ for all machines}} \tag{14.24}$$

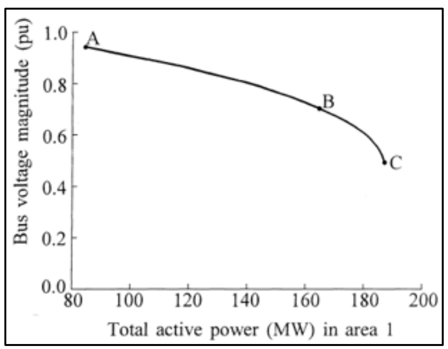
- Indicate, for each mode, which generators supply the most Q in response to an incremental change in system Q loading
- Important information regarding proper distribution of Q reserves

Voltage Stability Analysis

- Static analysis (Cont'd)

Illustration of modal analysis

- Consider Fig. 14.7
- Consider three operating conditions A, B, and C on the V-V curve of Fig. 14.4



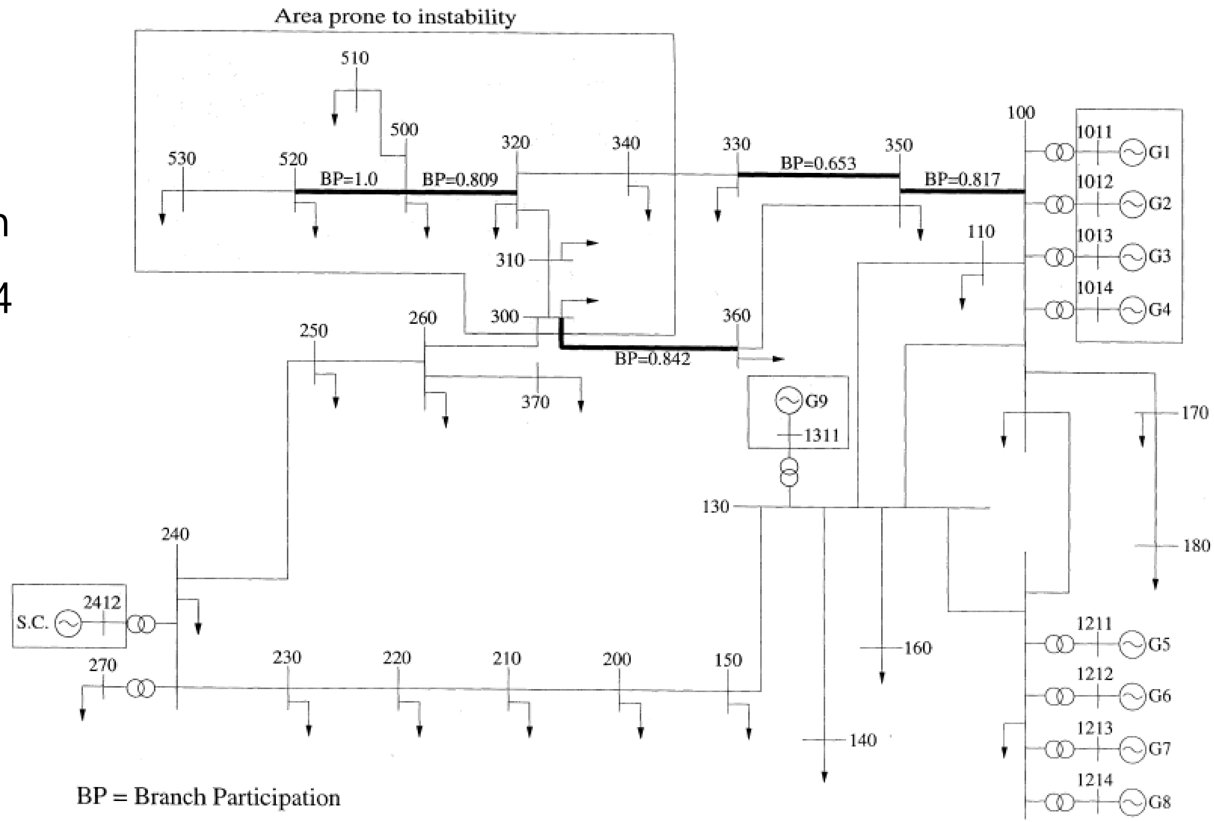


Figure 14.7 Buses and branches with high participation in the least stable mode

Voltage Stability Analysis

- Static analysis (Cont'd)

Illustration of modal analysis (Cont'd)

- Magnitudes of λ decrease as the system approaches instability
- At λ_1, point C, the system is on the verge of instability

Table 14.1 Five smallest eigenvalues

Operating Point	A	B	C
λ_1	0.3867	0.1446	0.0083
λ_2	1.0271	0.5550	0.3209
λ_3	2.4049	1.5133	0.9334
λ_4	4.1031	2.6280	1.8757
λ_5	4.2699	3.0209	2.3373

Voltage Stability Analysis

- Static analysis (Cont'd)

Illustration of modal analysis (Cont'd)

- Bus, branch, and generator participations for $\lambda = 0.0083$

Table 14.2 Bus, branch, and generator participations in the least stable mode for operating point C

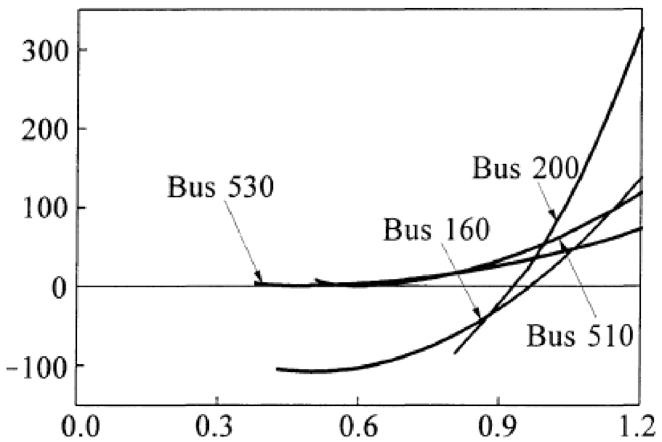
Bus Participation		Branch Participation		Generator Participation	
Bus	Participation	Branch	Participation	Bus	Participation
530	0.2638	500-520	1.0000	1311	1.0000
520	0.2091	300-360	0.8414	2412	0.2786
510	0.1025	100-350	0.8175	1011	0.2103
500	0.0941	320-500	0.8093	1014	0.2036
320	0.0482	330-350	0.6534	1013	0.2036
310	0.0319			1012	0.2036
300	0.0296				
340	0.0279				

Voltage Stability Analysis

- Static analysis (Cont'd)

Illustration of modal analysis (Cont'd)

- Interest to compare with Q-V characteristics shown in Fig. 14.5(c)
- Shows that buses 530 and 510 have zero Q margin
- Table 14.2 shows that these buses have high participation
- Comparison
  - Q-V analysis : Single-bus approach
  - Modal analysis : System-wide approach



Voltage Stability Analysis

- Determination of shortest distance to instability

Basic theory

- To find set of load P, Q increments whose vector sum is a minimum
- And when imposed on the initial condition, cause the Jacobian to be singular
- Power-flow equations in the form:

$$\mathbf{f}(\mathbf{x}, \boldsymbol{\rho}) = g\begin{bmatrix}\mathbf{V}\\ \boldsymbol{\theta}\end{bmatrix} - \begin{bmatrix}\mathbf{P}\\ \mathbf{Q}\end{bmatrix} = 0 \qquad (14.36)$$

where $\mathbf{x} = \begin{bmatrix}\mathbf{V}\\ \boldsymbol{\theta}\end{bmatrix}$ $\quad \boldsymbol{\rho} = \begin{bmatrix}\mathbf{P}\\ \mathbf{Q}\end{bmatrix}$: System vector and parameter vector
Both are $N = 2N_{PQ} + N_{PV}$ dimensional vectors

- Let $\mathbf{J_X}$ and $\mathbf{J_\rho}$ be the Jacobian matrices of $\mathbf{f}$ w.r.t. $\mathbf{x}$ and $\boldsymbol{\rho}$
- Let S denote the hypersurface in the N-dimensional parameter space such that $\mathbf{J_X}(\mathbf{x}_*, \boldsymbol{\rho}_*)$ is singular if $\boldsymbol{\rho}_*$ is a point on S

Voltage Stability Analysis

- Determination of shortest distance to instability

Basic theory (Cont'd)

- Initial point: $(\mathbf{x_0}, \boldsymbol{\rho}_0)$
- Find $\boldsymbol{\rho}_*$ on S such that the distance $k = |\boldsymbol{\rho}_* - \boldsymbol{\rho}_0|$ is a local minimum
- Assuming S is a smooth hypersurface near $\boldsymbol{\rho}_*$
- Normal vector to this hypersurface at $(\mathbf{x}_*, \boldsymbol{\rho}_*)$ is given by

$$\boldsymbol{\eta}_* = \mathbf{w}_* \mathbf{J}_\rho \qquad (14.37)$$

where $\mathbf{w}_*$ is the left eigenvector of $\mathbf{J}_x(\mathbf{x}_*, \boldsymbol{\rho}_*)$ corresponding to the zero eigenvalue.

$\boldsymbol{\eta}_*$ is normalized such that $|\boldsymbol{\eta}_*| = 1$

- Incrementally increasing $\boldsymbol{\rho}$ toward particular direction until $\mathbf{J_X}$ becomes singular; that is, $\boldsymbol{\rho}_* = \boldsymbol{\rho}_0 + k\boldsymbol{\eta} \qquad (14.38)$

Voltage Stability Analysis

- Determination of shortest distance to instability

Basic theory (Cont'd)

- Following procedure determines the vector $\boldsymbol{\eta}_*$ (for the shortest distance)

(1) Let $\boldsymbol{\eta}_0$ be an initial guess for the direction $\boldsymbol{\eta}_*$, $|\boldsymbol{\eta}_0| = 1$.

(2) Stress the system by incrementally increasing $\boldsymbol{\rho}$ along the direction of $\boldsymbol{\eta}_i$ until $\mathbf{J}_x$ becomes singular; that is, determine k_i, $\boldsymbol{\rho}_i$ and $\mathbf{x}_i$ so that $\boldsymbol{\rho}_i = \boldsymbol{\rho}_0 + k_i \boldsymbol{\eta}_i$ is on the surface S.

(3) Set $\boldsymbol{\eta}_{i+1} = \mathbf{w}_i \mathbf{J}_\rho$, and $|\boldsymbol{\eta}_{i+1}| = 1$.

(4) Iterate steps 1, 2, 3 until $\boldsymbol{\eta}_i$ converges to a value $\boldsymbol{\eta}_*$. Then, $\boldsymbol{\rho}_* = \boldsymbol{\rho}_0 + k_* \boldsymbol{\eta}_*$ is the corresponding equilibrium condition.

Voltage Stability Analysis

- Determination of shortest distance to instability

A simple radial system example

- Consider system shown in Fig. 14.8
- (14.36) gives:

$$\mathbf{f}(\mathbf{x}, \boldsymbol{\rho}) = \begin{bmatrix} 4V\sin\alpha - P \\ 4V\cos\alpha - 4V^2 - Q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{14.39}$$

$$\mathbf{x} = \begin{bmatrix} V \\ \alpha \end{bmatrix} \quad \text{and} \quad \boldsymbol{\rho} = \begin{bmatrix} P \\ Q \end{bmatrix}$$

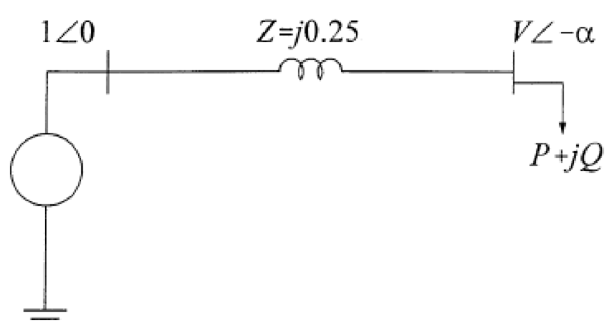


Figure 14.8 A simple radial system

Voltage Stability Analysis

- Determination of shortest distance to instability

A simple radial system example (Cont'd)

- Jacobian matrices are

$$\mathbf{J_x} = \begin{bmatrix} 4V\cos\alpha & 4\sin\alpha \\ -4V\sin\alpha & 4\cos\alpha - 8V \end{bmatrix} \quad (14.40)$$

$$\mathbf{J_\rho} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad (14.41)$$

- On the singular surfa$= 0$, $\det(\mathbf{J_X}) = 0$, that is,

$$\det(\mathbf{J_x}) = 16V - 32V^2\cos\alpha \quad (14.42)$$

or

$$V = \frac{1}{2\cos\alpha} \quad (14.43)$$

- Following expression describes the singular surface S :

$$P^2 + 4Q - 4 = 0 \quad (14.44)$$

Voltage Stability Analysis

- Determination of shortest distance to instability

A simple radial system example (Cont'd)

- Assume the initial condition as:

$$P_0 = 0.8 \qquad Q_0 = 0.4 \qquad V_0 = 0.8554 \qquad \alpha_0 = 13.52°$$

- Iterative process of finding the closest voltage instability point:

Table 14.3 Calculation of the shortest distance to voltage instability for the system of Figure 14.8

Iteration	Left Eigenvector $\boldsymbol{\eta}_i$	Distance to Instability (k_i)	P_i, Q_i
1	$[0.9725 \ \ -0.2331]^T$	1.0725	1.8430, 0.1500
2	$[0.6776 \ \ 0.7354]^T$	0.4173	1.0828, 0.7069
3	$[0.4869 \ \ 0.8735]^T$	0.4061	0.9977, 0.7541
4	$[0.4443 \ \ 0.8959]^T$	0.4024	0.9788, 0.7605
5	$[0.4405 \ \ 0.8977]^T$	0.4016	0.9769, 0.7605
6	$[0.4378 \ \ 0.8991]^T$	0.4015	0.9758, 0.7610

Voltage Stability Analysis

- Determination of shortest distance to instability

A simple radial system example (Cont'd)

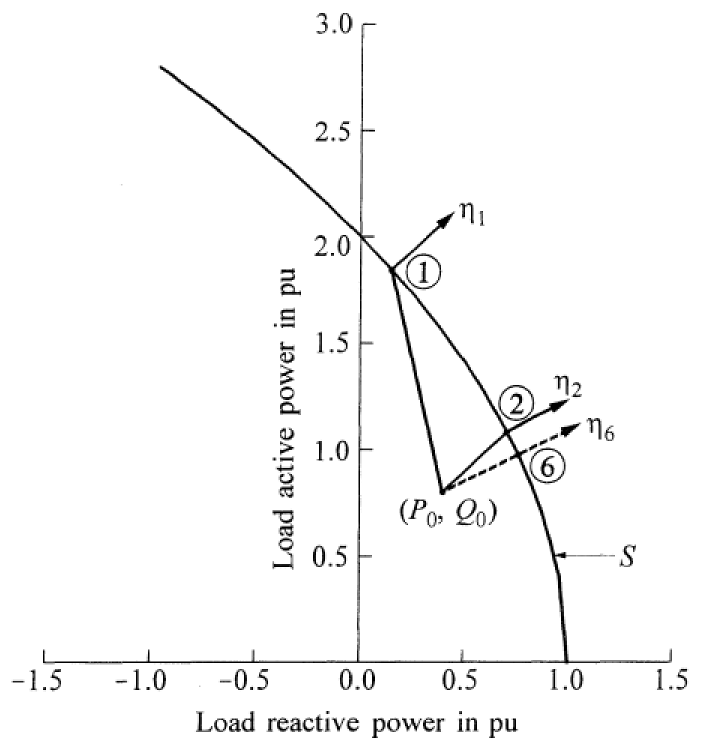


Figure 14.9 The singular surface S in the P-Q plane, and the convergence of the iterative process

※ In case of large system, the shape of the hypersurface is not known. We can expect that this process will find only a local minimum. It may be appropriate to use uniform loading for the initial direction.

Voltage Stability Analysis

- Determination of shortest distance to instability

General description of the procedure

(1) Increase load from $\mathbf{P}_0$, $\mathbf{Q}_0$ in some direction (the choice of the initial direction will be discussed later) until an eigenvalue of the Jacobian is practically zero. The load level $\mathbf{P}_1$, $\mathbf{Q}_1$ corresponding to this point is the stability limit. This point $\mathbf{P}_1$, $\mathbf{Q}_1$ lies on, or is extremely near, S.

Means instability

(2) For the conditions at $\mathbf{P}_1$, $\mathbf{Q}_1$, perform modal analysis and determine the left eigenvector of the *full Jacobian matrix*. The left eigenvector contains elements which provide the increments of MW and MVAr load for each bus. The eigenvector points in the shortest direction to singularity, which is therefore normal to S.

$$\boldsymbol{\eta}\Delta\mathbf{V} = \boldsymbol{\Lambda}^{-1}\boldsymbol{\eta}\Delta\mathbf{Q}$$

(3) Go back to the base case load level $\mathbf{P}_0$, $\mathbf{Q}_0$ and load the system again, but this time in the direction given by the left eigenvector found in (2). When S is reached, a new left eigenvector is computed.

(4) Again we return to the base case $\mathbf{P}_0$, $\mathbf{Q}_0$ and load the system in the direction of the new eigenvector given in (3). This process is repeated until the computed eigenvector does not change with each new iteration. The process will then have converged.

Voltage Stability Analysis

- The continuation power-flow analysis
  - Conventional power-flow algorithms are prone to convergence problems at operating conditions near the stability limit
  - Continuation power-flow analysis overcomes this problem

Basic principle

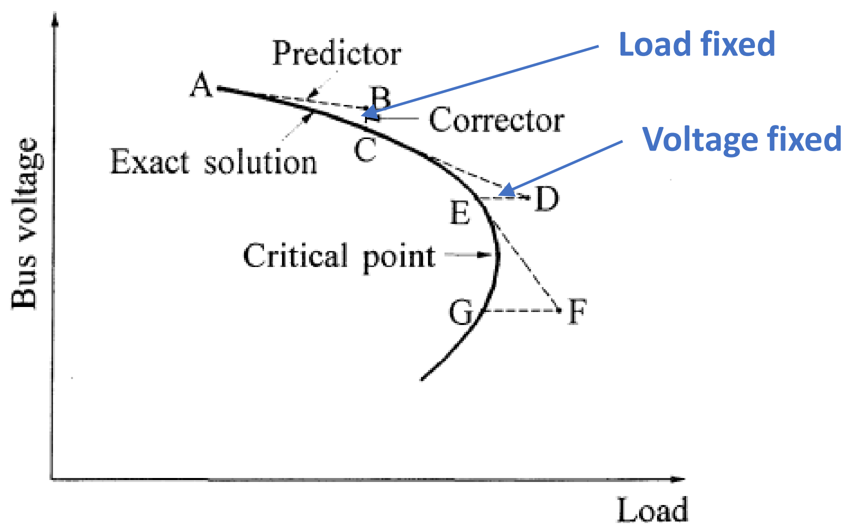


Figure 14.10 A typical sequence of calculations in a continuation power-flow analysis

Voltage Stability Analysis

- The continuation power-flow analysis

Mathematical formulation

- Similar to standard power-flow analysis except that load increase is added as a parameter
- Reformulated equation:

$$\mathbf{F}(\boldsymbol{\theta},\mathbf{V}) = \lambda\mathbf{K} \tag{14.45}$$

where λ is the load parameter
$\boldsymbol{\theta}$ is the vector of bus voltage angles
$\mathbf{V}$ is the vector of bus voltage magnitudes
$\mathbf{K}$ is the vector representing percent load change at each bus

$$\underset{\text{Base load}}{0} \le \lambda \le \underset{\text{Critical load}}{\lambda_{critical}}$$

- Equation may be rearranged as:

$$\mathbf{F}(\boldsymbol{\theta},\mathbf{V},\lambda) = 0 \tag{14.46}$$

Voltage Stability Analysis

- The continuation power-flow analysis (Cont'd)

Mathematical formulation: Predictor step

- Estimate next solution for a change in one of the state variables
- Taking derivatives of (14.46):

$$\mathbf{F_\theta} d\boldsymbol{\theta} + \mathbf{F_V} d\mathbf{V} + \mathbf{F_\lambda} d\lambda = 0$$

$$[\ \mathbf{F_\theta}\ \mathbf{F_V}\ \mathbf{F_\lambda}\ ] \begin{bmatrix} d\boldsymbol{\theta} \\ d\mathbf{V} \\ d\lambda \end{bmatrix} = 0 \qquad (14.47)$$

Setting one of the tangent vector components +1 or -1 imposes a non-zero value on the tangent vector and makes Jacobian nonsingular at the critical point.

- Since the unknown variable (λ) is added, one more equation is needed to solve
- This is satisfied by setting one of the components of the tangent vector to 1 or -1
- This component is referred to as the *continuation parameter*

$$\begin{bmatrix} \mathbf{F_\theta} & \mathbf{F_V} & \mathbf{F_\lambda} \\ & \mathbf{e_k} & \end{bmatrix} \begin{bmatrix} d\boldsymbol{\theta} \\ d\mathbf{V} \\ d\lambda \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \pm 1 \end{bmatrix} \qquad (14.48)$$

where $\mathbf{e_k}$ is all zero except kth element = 1 (corresponding to the continuation parameter)

Voltage Stability Analysis

- The continuation power-flow analysis (Cont'd)

Mathematical formulation: Predictor step (Cont'd)

- Initially, the load parameter λ is chosen as the continuation parameter
- And the corresponding component of the tangent vector is set to 1
- Next, the continuation parameter is chosen to be the state variable that has the greatest rate of change near the given solution
- Sign of its slope determines the sign of the corresponding component of the tangent vector
- Once the tangent vector is found, the prediction for the next solution is given:

$$\begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{V} \\ \lambda \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}_0 \\ \mathbf{V}_0 \\ \lambda_0 \end{bmatrix} + \sigma \begin{bmatrix} d\boldsymbol{\theta} \\ d\mathbf{V} \\ d\lambda \end{bmatrix} \tag{14.49}$$

- The step size σ is chosen so that a power flow solution exists with the specified continuation parameter
- If a solution cannot be found, reduce the step size

Voltage Stability Analysis

- The continuation power-flow analysis (Cont'd)

Mathematical formulation: Corrector step

- The original set of equations is augmented by one equation:

$$\begin{bmatrix} \mathbf{F}(\boldsymbol{\theta},\mathbf{V},\lambda) \\ \underbrace{x_k}_{\text{Selected as the continuation parameter}} - \underbrace{\eta}_{\text{Predicted value of } x_k} \end{bmatrix} = [\mathbf{0}] \qquad (14.50)$$

  Introduction of the additional equation makes the Jacobian non-singular at the critical point

- The tangent component λ (i.e. $d\lambda$) is
  - Positive for upper portion of *V*-*P* curve
  - Zero at the critical point
  - Negative beyond the critical point

See Fig. 14.10

Continuation parameter	Corrector line
Load increase	Vertical
Voltage magnitude	Horizontal